COLORADO MATHEMATICS TEACHER - FALL 2016

The Official Publication of Colorado Council of Teachers of Mathematics

CCTM
COLORADO COUNCIL OF TEACHERS OF MATHEMATICS

Procedural Fluency & Mathematical Discourse

CONTENTS & FEATURES

The CMT presents a variety of viewpoints. Unless otherwise noted, these views should not be interpreted as official positions of CCTM or CMT.

From the Editor's Desk

Sandie Gilliam, Editor

I am delighted to announce that your very own *Colorado Mathematics Teacher* has won the 2016 NCTM Publication Award for best newsletter, chosen from applications by affiliates from around the country. The evaluators looked at mathematics content/pedagogy and news, value to membership, and the quality and readability of the writing. I can't tell you how pleased and honored I am with this award. It reflects the hard work and dedicated professionalism of the authors, board members, editorial panelists, and contributors like you. It's your newsletter, and you deserve to be proud of the award as well.

When I volunteered for this job in 2012, I was a bit apprehensive. I knew how to teach mathematics, but had no knowledge of the publishing world. I never considered myself an English expert and still don't, but thankfully work with my editorial board and my family. My first *CMT* edition was black and white and was sent to a typesetter who charged for online formatting. My goal soon became to publish in color and take the newsletter in-house. Thanks to the wonders of Adobe InDesign and a special volunteer who has leaned it well, that goal was realized two years ago.

My goal on the CCTM Board is to: give back for all that was given to me over a math teaching career that began in 1975, and to pass along to others the professional knowledge learned along the journey. Many experiences framed this journey: 32 years teaching in a secondary math classroom, followed by 10 years working with pre-service and experienced teachers; speaking opportunities and assessment item writing (both locally, state-wide, and nationally); portfolio writing and video opportunities for National Board Certification and PAEMST Award processes; opportunities to learn from and work directly with Ruth Parker, Deborah Ball, and Jo Boaler; leadership learning as an NCSM board member; and 40+ years as an NCTM member—where conference attendance, journal reading, and an opportunity to write a chapter in NCTM's book, *Mathematics for Every Student, Responding to Diversity, Grades 9-12* have given me opportunities to both learn and give back. From a young teacher looking up to the NCTM presidents for words of wisdom to an experienced teacher dialoguing with them in person, to a veteran teacher receiving an award from a colleague (NCTM President Diane Briars), my moment that day in April was very reflective—and awesome!

One never knows where their journey as a teacher takes them, and what experiences will be presented along the way. Should you jump at opportunities and extend your current responsibilities, establish desired accomplishments for waypoints during your career, actively pursue your dreams, and/or be satisfied with your status quo?

Have a great new beginning to your school year. No matter where you are in your teaching career, giving back feels really good! Whose teaching journey might you impact this year?

The NCTM Affiliate Services Committee (ASC) awarded the Colorado Council of Teachers of Mathematics' "Colorado Mathematics Teacher (CMT)" 2016 Publication Award for Outstanding Newsletter. NCTM President Diane Briars presented the plaque to CMT Editor Sandie Gilliam at the NCTM Annual Conference in San Francisco.

Math Discourse

Joanie Funderburk, CCTM President

Although it was nearly 27 years ago, I'll never forget the first time a school administrator observed me teaching in my high school math classroom. He said, "Joanie, you have done a great job of getting your students to talk during your Geometry class. Unfortunately, they aren't talking about Geometry." Although I was disheartened, I knew he was right, and I set out to shape my chatty students into articulate math speakers. I had some small successes that first year, and ever since then, articles, workshops, and PLC conversations about math discourse have always caught my attention.

In NCTM's *Principles to Actions,* we are given the guidance of the eight Mathematics Teaching Practices. These instructional strategies are proven effective in supporting student learning of math, and *Principles to Actions* helps us know how to work these strategies into our practice. Here, we are reminded that meaningful mathematical discourse serves to build shared understanding of mathematical ideas. In other words, the goal is not to have students just speak about math, but to use their discussions to further enhance how and what they think about the important mathematical concepts they are learning.

For instance, when each student in a small group presents his or her solution strategy to the rest of the group, one at a time and without discussion, the students are speaking about math. However, it is unlikely that their learning is impacted by this talk because they aren't asking questions, analyzing, rephrasing, or agreeing or disagreeing with what they've heard. If instead, after each student presents, the other students ask for clarification or make connections to their own strategies, they can build a deeper understanding of the mathematics being discussed. As an added bonus, this type of discourse builds students' confidence, enhances relationships, and supports a culture of learning in the classroom.

During my first few years of teaching, I had a hard time knowing how to make the right changes in my instructional strategies and classroom policies and procedures that would allow student discussions to be productive and supportive of learning. Now, however, we have great information available to us, such as Parrish's *Number Talks* books, Smith and Stein's *Five Practices for Orchestrating Productive Mathematical Discussions*, and of course, NCTM's *Principles to Actions*. These resources and others point to the importance of choosing meaningful tasks for students to discuss, using probing questions that ask for justification, and helping students learn how to question, listen, and learn from one another in addition to their teacher.

An additional benefit of meaningful student discourse is its role as a formative assessment. When our classrooms have structures in place where students talk about their thinking, respond to one another, and make connections out loud, we can better identify their pre- and misconceptions and know what they need next in their developing mathematical thinking. Discourse allows us to differentiate our instruction and select our next instructional moves to be sure all students are learning and progressing.

As you kick-off the 2016–17 school year, this issue of *Colorado Mathematics Teacher* is rich with ideas for you to grow in your discourse facilitation. Be sure to share these articles, the books mentioned above, and other tools in your PLCs to facilitate your own meaningful mathematical discourse.

I hope the new school year holds great experiences for you and your students, and I look forward to seeing you September 22–23 at our annual conference!

EQUITY: MATH FOR ALL

Committing to Equity: Going Beyond Teaching Mathematics

Nora Ramirez, President, Arizona Association of Teachers of Mathematics

Editor's Note: Before each of our CCTM fall conferences, an article written by one of the main presenters is featured so that members can glean a sense of what the speaker values and session attendees will be delving further into.

As I think about the new school year beginning, I know that teachers review the Academic Mathematics Standards they are responsible for teaching, prepare their classrooms, and make detailed plans. They reflect on what was learned in the past years from both students and colleagues, what insights and new ideas were gathered from professional development attended or research read, and consider the classroom procedures they want to put in place for all students to be successful. After all, it is a new year!

Quality standards, an aligned curriculum, and effective teaching practices are necessary elements to achieve student learning in mathematics. If we want to maximize learning for each student in our classrooms, we must address the essential elements necessary for successful mathematics programs. They are: a commitment to access and equity, a powerful curriculum, appropriate tools and technology, meaningful and aligned assessment, and a culture of professionalism (NCTM, 2014). A commitment to access and equity requires not only believing that all students can learn mathematics but also employing actions aligned to that belief. There are many actions, all important and necessary, that will impact the goal of achieving equity in- and access to- school mathematics. I present a few actions, some of which are not often targeted because one might not think them directly related to the teaching and learning of mathematics. I do!

High Expectations and Support

All students are capable of learning mathematics. Unfortunately, not everyone nor every teacher believes this. A student's race, class, culture, language and gender, more often than not, negatively influences the beliefs about a student's ability to learn. Beliefs, policies and practices need to be examined by all educators and actions taken to ensure that schools, classrooms, curriculum, instruction, and student placement deliberately give each student an opportunity to learn. It is essential that mathematics tasks be engaging and accessible to all students, yet maintain rigor. If not, tasks should be adapted or replaced with a clear focus on the grade-level mathematics that students are expected to learn.

In addition, as educators, we need to be aware of our language and eliminate (or challenge) any deficit language that we say (or hear) in conversations. Labels such as "low kids" and "high kids" are unjust as they privilege some and marginalize others (NCSM & TODOS, 2016). Grouping students by perceived ability results in inequitable mathematical experiences with the "low kids" traditionally not being given the same opportunity to learn mathematics as the "high kids". Tracking and grouping students into low-level groups, as well as labeling students as "slow," have damaging effects on the expectations of both teachers and students and should be eliminated.

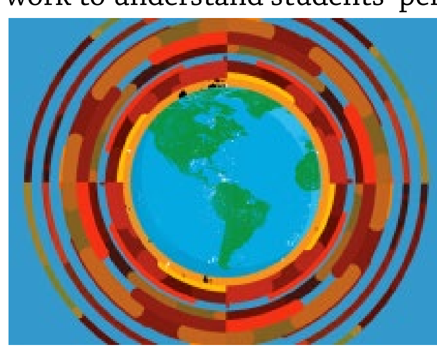
All students, including English language learners (ELLs), need support while learning mathematics. We expect students to engage in mathematics, reason, explain their thinking, and critique the reasoning of others. Students whose first language is

other than English need processing time and support to speak aloud, engage in discourse, produce correct sentence structure, and use precise mathematical language (Celedón & Ramirez, 2012). For example, a teacher of a second grade classroom of ELL students was concerned because her students would not speak aloud, thus not engaging in discourse. To address this, the students became actors in a play, learned their parts, and produced language aloud in front of others. The play involved them in speaking memorized lines, so the cognitive demand was low, yet the emotional demand was high as they feared making mistakes (Ramirez & Tapetillo, forthcoming). In this case, the teacher had high expectations, raised the students' comfort levels in speaking aloud in front of others, and developed a classroom culture where students openly shared, asked questions, and discussed mistakes. The students did not learn any mathematics by participating in the play, but they developed the skills to participate in a mathematics discourse community, thus affecting their ability to learn mathematics.

Supporting students can vary. Teachers support students while exploring tasks by asking questions, helping students consider what they know, fostering the use of representations, and not thinking for the students. They support students in developing conceptual understanding using the Concrete-Representational-Abstract model of instruction when applicable and guide students to make connections between these stages and various representations. Teaching ELLs requires on-going support so they can learn English while learning mathematics and be active participants in a mathematics discourse community. To support students in problem solving, teachers can focus attention on understanding a problem situation, at times giving no numbers in a problem, to enable students to attend to what is happening in the situation, and consider what students are asked to find without being distracted with numbers. The use of anchor charts and interactive mathematics notebooks benefits students in conceptualizing mathematical ideas, using precise language, utilizing multiple tools and strategies, and applying mathematics. In addition, on-going classroom-based assessments support student learning by allowing the teacher to determine what students know, build on that knowledge, and move students forward mathematically. Instead of asking why doesn't a particular student know a particular concept, we should be asking what the student knows and understands, and how we can use that understanding to facilitate further learning?

Building Relationships

In addition to having high expectations and supporting students learning mathematics, it is important to recognize and act on the need to build relationships with students and build on their culture, conditions, and language (NCTM, 2014). In many of our schools the teaching population is not representative of the student population. Thus, in order to be responsive to students' background and experiences, educators need to interact with students, listen to students, learn about their community, and work to understand students' perspectives.



In a recent study of an intervention to improve teacher-student relationships, preliminary research indicated that when teachers saw similarities between themselves and their students, the achievement gap was reduced by more than 60 percent. While more research is needed to generalize the findings to a larger population, one cannot ignore them completely if we are concerned about making a difference in student achievement (Gehlbach & Robinson, 2016). Thus teachers may consider actions or activities to determine what similarities there are between themselves and their students, thereby taking a step in building relationships.

In the spring of 2010, Arizona passed a highly controversial and tough immigration bill, Senate Bill 1070. Aware of her students' personal struggles and learning distraction with the passage of this bill, a 5th grade teacher recognized the need to pull the class together and give them time to ask questions, discuss their concerns and feelings, and let them know that she cared about them. After the discussion and clarification of the law, the students were ready to learn. This exemplifies a teacher who is concerned about her students' personal and emotional needs and understands the importance of building relationships with her students. Now consider the racial turmoil that occurred this summer in the United States, and reflect on how it might be affecting students emotionally and how it might affect teacher-student relationships. I ask you to consider steps that a teacher or other educators can take to open up dialogue with students to unfold their concerns and understand their perspectives. Granted, immigration and racial turmoil may not be related to mathematics, but open class discussions let students know that their teachers are interested in things that matter to them, thus affecting student interest and desire to learn.

Building Student Self-Confidence

For some, their lack of success in mathematics is related to their lack of confidence in themselves as a learner of mathematics. Students, and often their parents, believe that one is born with or without a math brain. Some educators even foster this idea. These beliefs are unproductive and need to be addressed. Not only should educators develop a growth mindset, they should facilitate their students in developing one also. A growth mindset stresses that learning is the result of effort, that all students are capable of learning mathematics, and that mistakes cause one's brain to grow (Boaler, 2016).

Having a growth mindset works in parallel with having high expectations and believing that all students are capable of learning mathematics. Developing a growth mindset in our students will build their self-confidence as mathematics learners. It is an intervention that is inexpensive, requiring both teacher and student learning, and a teacher's dedication to continually use growth mindset language and reinforce growth mindset thinking in students.

Conclusion

In order to implement the eight Mathematical Teaching Practices effectively, there must be a commitment to access and equity. High expectation of students, classroom support to reach those expectations, building relationships and addressing student self-confidence (even though they may be without a mathematical context) are important if we are to affect students access to- and equity in- mathematics.

References

Arizona S.B. 1070 Immigration; Law Enforcement; Safe Neighborhoods. (2010, January 15). Retrieved from http://www.azleg.gov/legtext/49leg/2r/summary/s.1070pshs.doc.htm

Boaler, J. (2016). *Mathematical Mindsets: Unleashing Students' POTENTIAL Through Creative Math, Inspiring Messages and INNOVATIVE TEACHING*. San Francisco: Jossey-Bass.

Celedón-Pattichis, S. & Ramirez, N. (2012). *Beyond Good Teaching: Advancing Mathematics Education for ELLs*. Reston, VA: National Council of Teachers of Mathematics.

Gehlbach, H. & Robinson, C. (2016, August 3). Creating Birds of a Feather: The Potential of Similarity to Connect Teachers and Students. *American Enterprise Institute*. Retrieved from http://www.aei.org/publication/creating-birds-of-a-feather-the-potential-of-similarity-to-connect-teachers-and-students/

National Council of Supervisors of Mathematics (NCTM) & TODOS: Mathematics for All. (2016). *Mathematics Education Through the Lens of Social Justice: Acknowledgement, Actions and Accountability*. Retrieved from: http://www.todos-math.org/socialjustice

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to Action: Ensuring Math-*

ematical Success for All. Reston, VA: National Council of Teachers of Mathematics.

Overview for Interactive Notebook. (n.d.). Retrieved August 23, 2016, from http://upstagereview.org/ClassroomArticles/interactive notebook.pdf

Ramirez, N. & Tapetillo, S. H. (forthcoming). Supporting ELLs in a Discourse Community. In Celedón-Pattichis, S., White, D. Y., & Civil, M. (Eds.), *Access and Equity: Promoting High Quality Mathematics in Grades PreK-2*. Reston, VA: National Council of Teachers of Mathematics.

Sowinski, R. M. (2016, August 8). Summer Schooled Us: Broken Bonds and the First Day. Retrieved from http://www.huffingtonpost.com/entry/white-teachers-first-day-of-school_us_579b923ee4b00e7e269f2244

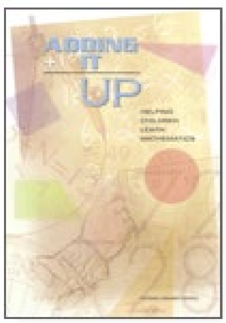
FOCUSING IN ON

Procedural Fluency

Sandie Gilliam, CMT Editor

ONE OF THE TWO FOCI for this issue, from *Principles to Actions (PtA)*, is **build procedural fluency from conceptual understanding** (NCTM, 2014). This teaching practice states that: *"Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems."*

I attended Diane Briars session, *Supporting Teachers in Building Procedural Fluency from Conceptual Understanding*, at the NCSM Annual Conference in April, and wanted to share some of her main points.



What is procedural fluency? It is one of the five strands of mathematical proficiency listed in Adding It Up, a 2001 National Academies Press document. Defined as "the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately," procedural fluency is interwoven with the conceptual understanding strand.

Watch the videos from The Marilyn Burns Math Reasoning Inventory—Alan (1000 – 98), Alan (100 – 18), Ana (1000 – 98), Ana (99 + 17), and Marissa (295 students, 25 on each bus)—to determine which of the students demonstrate procedural fluency and jot down evidence for your response.

Procedural fluency can be further explained as:

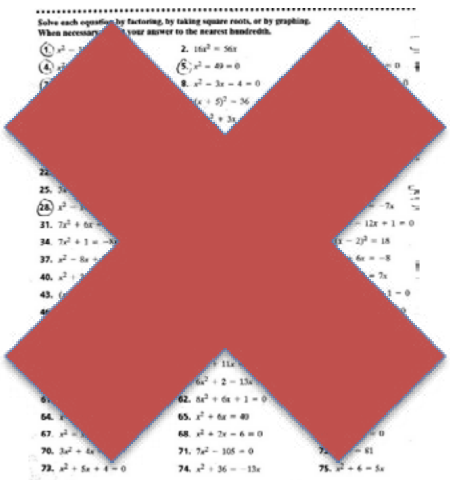
- **Efficiency—**can carry out easily, keep track of sub-problems, and make use of intermediate results to solve the problem.
- **Accuracy—**reliably produces the correct answer.
- **Flexibility—**knows more than one approach, chooses appropriate strategy, and can use one method to solve and another method to double-check.
- **Appropriately—**knows when to apply a particular procedure.

In what ways do these further support your evidence from the above videos?

If you haven't already seen the *Principles to Actions Toolkit*, the Case of "Mr. Harris and the Band Concert Task," check it out. This free NCTM module will provide you with the presentation, presenter notes, and required materials to support your professional learning in analyzing artifacts of teaching (e.g., mathematical tasks, narrative and video cases, student work samples, vignettes) and abstracting from the specific examples general ideas about how to effectively support student learning. In the "Band Concert Task," pay particular notice to the section on *building procedural fluency from conceptual understanding*. (Other free modules are available, as well as additional ones that NCTM members can access.)

What is meant by 'standard algorithm'? Fuson & Beckmann in the 2013 *NCSM Journal* (p. 14) state, "In mathematics, an algorithm is defined by its steps and not by the way those steps are recorded in writing. With this in mind, minor variations in methods of recording standard algorithms are acceptable." Why as I work with pre-service and experienced teachers, are many still providing their students with ONLY the same ONE algorithm they learned themselves as a student? Why are many parents insisting on using this same ONE algorithm with their children? Briars explored multiple variations of algorithmic methods: whole number multiplication, equation solving, dividing fractions, and finding the unknown in proportions in her presentation—only with conceptual understanding a critical part.

Many teachers read this teaching practice—build procedural fluency—and think, “Nothing new here; I’ve been doing this for decades!”

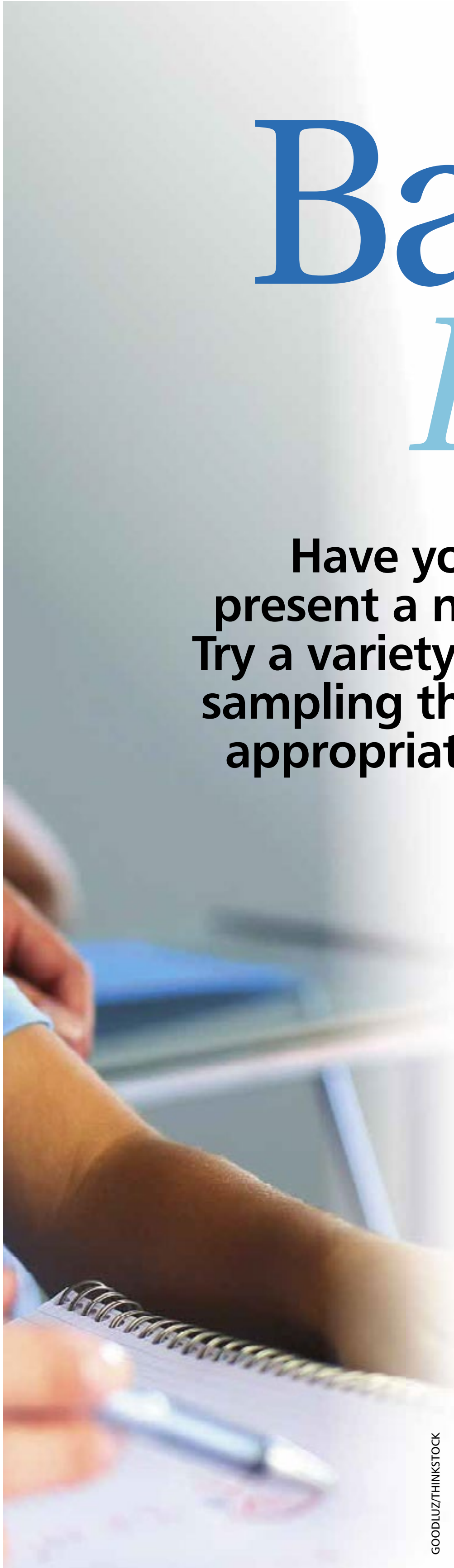


Is there really nothing new? Consider the following from *PtA:*

What are teachers doing?	What are students doing?
Providing students with opportunities to use their own reasoning strategies and methods for solving problems.	Making sure that they understand and can explain the mathematical basis for the procedures that they are using.
Asking students to discuss and explain why the procedures that they are using work to solve particular problems.	Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.
Connecting student-generated strategies and methods to more efficient procedures as appropriate.	Determining whether specific approaches generalize to a broad class of problems.
Using visual models to support students’ understanding of general methods.	Striving to use procedures appropriately and efficiently.
Providing students with opportunities for distributed practice of procedures.	

Words of wisdom from NCTM President Diane Briars!

Assessing Basic Fact *Fluency*



GOODLUZ/THINKSTOCK

Have you had it with timed tests, which present a number of concerns and limitations? Try a variety of alternative assessments from this sampling that allows teachers to accurately and appropriately measure children's fact fluency.

By Gina Kling and Jennifer M. Bay-Williams

Think about how you assess reading fluency. Does your assessment plan involve listening and observing as children read as well as asking reading comprehension questions? Now imagine what you might learn about students' reading fluency if you used *only* timed quizzes. How would your confidence in your assessment change?

Formative assessments—including observations, interviews, performance tasks, and journaling—have become common practice in many classrooms, with a recognition that by using different ways to assess children, we gain a more comprehensive, accurate picture of what they know, what they do not know, and their misconceptions. These data are then used to design instruction accordingly (Wiliam 2011). Yet, in spite of this trend in other areas of education, timed, skill-based assessments continue to be the prevalent measure of basic mathematics facts achievement. As a result, many rich opportunities for assessing basic fact fluency are lost. In this article, we share a variety of ways to formatively assess basic fact fluency. We define fluency, raise some issues related to timed testing, and then share a collection of classroom-tested ideas for authentic fact fluency assessment.

Defining fluency

A variety of interpretations exist for what procedural fluency (in general) and basic fact fluency (specifically) mean. Fortunately, recent standards, research, and reports provide a unified vision of what these terms signify. The Common Core State Standards for Mathematics (CCSSM) document describes *procedural fluency* as "skill in carrying out procedures *flexibly*, *accurately*, *efficiently*, and *appropriately*" (CCSSI 2010, p. 6). Likewise, Baroody (2006) describes basic fact fluency as "the efficient, appropriate, and flexible application of single-digit calculation skills and . . . an essential aspect of mathematical proficiency" (p. 22). These definitions reflect what has been described for years in research and standards documents (e.g., NCTM 2000, 2006; NRC 2001) as well as CCSSM grade-level expectations related to basic facts (see **table 1**).

Notice that the CCSSM expectations use two key phrases; the first is to *fluently* add and subtract (or multiply and divide), and the second is to *know from memory* all sums (products) of two one-digit numbers. To assess basic fact *fluency*, all four tenets of fluency (flexibility, appropriate strategy use, efficiency, and accuracy) must be addressed. Additionally, assessments must provide data on which facts students *know from memory*. Timed tests are commonly used to supply such data—but how effective are they in doing so?

TABLE 1

Past mathematics documents—as well as current standards, studies, and reports—offer a unified vision of what procedural fluency means.

CCSSM standards that address fluency and mastery with basic facts (italics added)

Kindergarten K.OA.A.5	*Fluently* add and subtract within 5
Grade 1 1.OA.C.6	Add and subtract within 20, *demonstrating fluency* for addition and subtraction within 10. *Use strategies* such as counting on; making 10; decomposing a number leading to a 10; using the relationship between addition and subtraction; and creating equivalent but easier or known sums
Grade 2 2.OA.B.2	*Fluently* add and subtract within 20 using *mental strategies* (refer to 1.OA.6). By the end of grade 2, *know from memory* all sums of 2 one-digit numbers
Grade 3 3.OA.C.7	*Fluently* multiply and divide within 100, using *strategies* such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 / 5 = 8) or properties of operations. By the end of grade 3, *know from memory* all products of 2 one-digit numbers.

Limitations and risks of timed mathematics tests

Timed tests offer little insight about how flexible students are in their use of strategies or even which strategies a student selects. And evidence suggests that efficiency and accuracy may actually be negatively influenced by timed testing. A study of nearly 300 first graders found a negative correlation between timed testing and fact retrieval and number sense (Henry and Brown 2008). Children who were frequently exposed to timed testing demonstrated *lower* progress toward knowing facts from memory than their counterparts who had not experienced as many timed tests. In fact, growing evidence suggests that timed testing has a negative impact on students (Boaler 2012, Henry and Brown 2008, Ramirez et al. 2013). Surprisingly, the anxiety that many children experience over timed testing is unrelated to how well they do on the tests. Even high-achieving children share such concerns as, "I feel nervous. I know my facts, but this just scares me" (Boaler 2012). Math anxiety appears as early as first grade, and this anxiety does not correlate with reading achievement (Ramirez et al. 2013). In other words, children's anxiety is specific to mathematics, not general academic work. And the struggling learner is not the only one who experiences anxiety: Ramirez and his colleagues found that children demonstrating a higher use of "working memory" (i.e., those who tended to use mathematical strategies that were more sophisticated) experienced the most negative impact on achievement as a result of math anxiety. Thus, it appears that some of our best mathematical thinkers are often those most negatively influenced by timed testing.

Fortunately, children can learn facts effectively without the use of timed testing. In a longitudinal study of twenty second graders, Kling found that without any timed testing or other rote fact activities, by the end of the year, the children demonstrated automaticity with addition facts (solved within 3 seconds) 95 percent of the time. Interestingly, the children performed strategies (e.g., making ten or near doubles) so quickly that it was impossible to distinguish between highly efficient strategy application and "knowing from memory." Since the beginning of first grade, fact practice for these children had involved (a) activities within textbook lessons (b) weekly fact games and (c) activities such as Quick Images with ten frames that were used to foster discussion around strategies (see Kling 2011, Bay-Williams and Kling in press). This research suggests that timed assessments and drill may not be necessary for children to achieve fact mastery.

If timed mathematics assessments have questionable value and potentially negative psychological, emotional, and educational impact, why are they still so frequently used? We commonly hear three reasons. First, fluency is interpreted as synonymous with speed. We have already addressed that fluency is more comprehensive than speed. Second, some feel that timed tests prepare children for high-stakes tests. The research shared here convincingly shows it may do the opposite. Third, timed tests are the only assessments widely available for assessing fluency of basic facts. As we seek to rectify this last concern, the remainder of this article shares methods for more comprehensively and appropriately assessing students' basic fact fluency.

Using formative assessment strategies

With an eye on the aspects of fluency (accuracy, efficiency, flexibility, and appropriate strategy selection), we can use various assessment strategies to see what students know (and do not know) and determine what our next instructional steps might be. All are approaches we have used with children in grades 1–4, and when used in combination with one another, these methods provide a comprehensive picture of a student's level of fact mastery.

1. Interviews

Interviews provide the extraordinary opportunity to hear children explain what they know about a topic in a discussion format, during which teachers can ask follow-up and clarifying questions (Hodges, Rose, and Hicks 2012; Van de Walle et al. 2014). The insights gained from listening to a child can be invaluable for planning individualized instruction or interventions. And insights from a selection of students can inform instruction for the whole class. Consider which aspects of fluency to address using the questions posed in each of the sample interview protocols (see **fig. 1**). We see the interview as a quick way to get at all four categories of fluency (as well as to see if a student just knows

FIGURE 1

Below are protocols for student interviews, which are a way to quickly assess all four categories of fluency and see if a student just knows a fact. Insights from a selection of interviews can inform instruction for the whole class.

Protocol A. Assess **fluency**

1. Write 4 × 5 on a card [*point at card*].
   What does 4 × 5 mean?
2. What is the answer to 4 × 5?
3. How did you find the answer to 4 × 5?
   Could you find it another way?
4. If your friend was having trouble remembering this fact, what strategy might you suggest to him or her?

Protocol B. Assess **flexibility** and **strategy selection**

1. What is 8 + 5?
2. How can you use 8 + 2 to help you solve 8 + 5?

OR

1. How can you use 3 × 7 to solve 6 × 7?

Protocol C. Assess **use of appropriate strategy**
(adapted from Henry and Brown 2008)

Probes
What is 7 + 8?
How did you figure it out? [*Ask regardless of how quickly or accurately they solve the fact*.]

Codes

R	= Recall
A	= Automatic (within 3 seconds)
M10	= Making 10 Strategy
ND	= Near Doubles Strategy
D	= Some other derived fact strategy
CO	= Counting on
CA	= Counting all
MCA	= Modeling and counting all

the fact). *Accuracy* is assessed as soon as the student responds, and *efficiency* is observed on the basis of how long it takes a student to solve the fact. *Flexibility* and *appropriate strategy selection* are addressed by such follow-up prompts as, "How did you figure it out?" or "How could you use this strategy to solve this fact?" Codes, such as those suggested in Protocol C, can facilitate recording during an interview (see **fig. 1**).

Interviews need not be one-on-one, sit-down events. They can be quick exchanges in the midst of other activities. For example, as students are lining up, ask, "Aaron, what is six times seven? How did you figure it out?" Furthermore, interviews have an added benefit of allowing students the opportunity to self-correct. For example, during interview assessments with thirty-eight beginning first graders, Kling found that 54 percent of the time children self-corrected incorrect answers as they explained how they figured out the fact. Furthermore, the children's self-reported strategies (i.e., "I counted on my fingers" or "I just knew it") were consistent with what the interviewer was able to observe 97 percent of the time. This suggests the potential of interviews as highly reliable and informative assessment tools.

2. Observations

Observation is a natural part of teaching, and recognizing which strategies students know can supply valuable insights to help support students as they learn new strategies and tackle unknown facts. To create organized and accurate records of observations, a list of students and facts can be attached to a clipboard (see

TABLE 2

Codes can facilitate recording during an interview.

(a) Use an accuracy table to review students' progression with addition facts.

	Within 5	Foundational facts			Within 10	Within 20
Name/facts		0, 1, 2	Combinations that make 10	Doubles		
Nicholas						
Kayla						
Cynthia						
Robbie						
⋮						

(b) A table can show the frequency of addition fact strategy use at a glance.

Name/strategies	1 more/ 2 more	Combinations that make 10	Making 10	Doubles	Find 5s	Applies commutativity
Nicholas	+			+	+	+
Kayla	+			+	+	
Cynthia		+		+		+
Robbie	+	+	+			+
⋮						

MEAGHAN GORZENSKI

As students turn over cards, observe to see and hear how *efficient* each student is as well as whether he or she chose an *appropriate strategy* or just knew.

table 2a); or a list can be tracked on the basis of which strategies students use (see **table 2b**). Equipped with these charts, you can observe as students engage in facts games, such as mathematized versions of classic games of War, Go Fish, Concentration, Old Maid, and Memory (for other games to teach and review basic facts, see Forbringer and Fahsl 2010; Kamii and Anderson 2003; Van de Walle, Karp, and Bay-Williams 2013; Kling 2011; Bay-Williams and Kling, in press; Kling and Bay-Williams, in press).

A critical aspect of meaningful use of games is to ask students to tell their teammates both the answer *and* how they found it. As students turn over cards, observe to see and hear how *efficient* each student is as well as whether he or she chose an *appropriate strategy* or if they just knew. The teacher might observe, for example, that many students are more *efficient* at solving 5 + 3 than they are at 3 + 8. These students may "just know" facts within ten but may apply strategies for the facts that have sums over ten. Such insights gained through observation can help the teacher select appropriate activities for continued learning and practice.

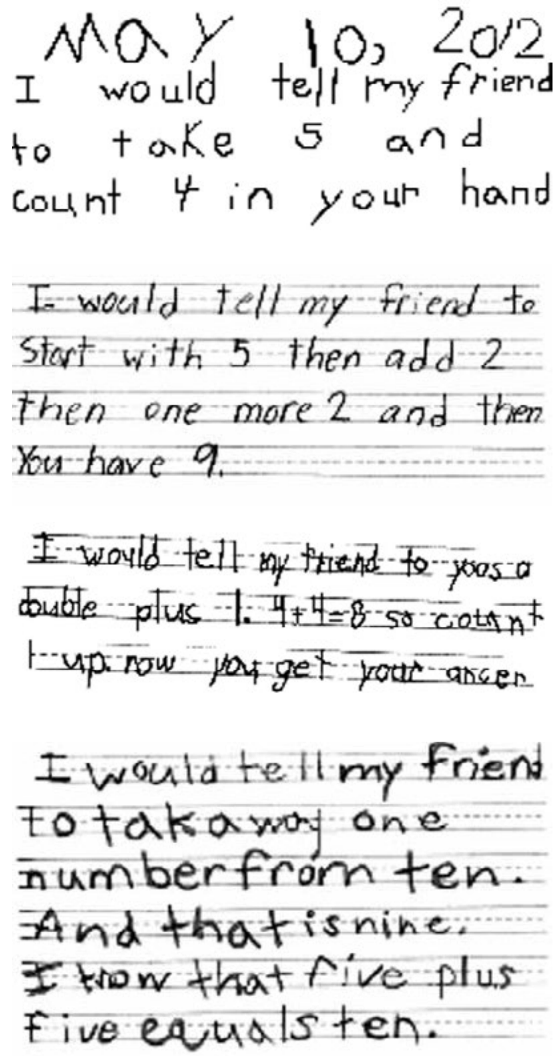
To enhance opportunities for assessing students during game play, consider having groups rotate through centers, stationing the teacher at one center and using probing questions, such as "How did you figure that out?" or "Are there any other ways you could figure it out?" One first-grade teacher had the following to say after using an observational checklist to formatively assess her students:

> This is an important tool that provides a more comprehensive check of which specific strategies a student has successfully mastered toward developing fluency with their basic facts. CCSSM provide specific strategies that students are expected to understand and use, and the chart provides me the opportunity to learn which strategies are being used effectively and where there are opportunities for further instruction and practice.

FIGURE 2

Various responses to a journal prompt illustrate the strategies that first graders used and reveal which children were able to appropriately select and explain an efficient strategy for the task.

If your friend did not know the answer to 4 + 5, how could he figure it out?



3. Journaling

Writing provides an excellent opportunity to assess flexibility and understanding of strategy selection and application. Children at any grade level can find ways to incorporate pictures,

> *Writing provides an excellent opportunity to assess flexibility and understanding of strategy selection and application. Children at any grade level can find ways to incorporate pictures, words, and numbers to communicate their strategies.*

words, and numbers to communicate their strategies. For example, **figure 2** shows a variety of first graders' responses to the journal prompt, "If your friend did not know the answer to 4 + 5, how could he figure it out?" Carefully review the responses, considering what they illustrate about the strategies used by the children. In contrast to what can be learned from a child's answer to 4 + 5 on a timed test, these samples offer rich opportunities to recognize which children can appropriately select and explain an efficient strategy for the task. This is important for deepening strategy understanding and also is reflected in the expectations of CCSSM and the related, forthcoming assessments. For example, Smarter Balanced Assessment Consortium (SBAC) lists the following as "evidence required" for grade 3. Note the application of strategies inherent in these expectations. The student—

- multiplies and divides facts *accurately*;
- multiplies and divides facts *using strategies*, such as the relationship between multiplication and division or properties of operations; and
- uses multiplication and division facts (SBAC 2012) (emphasis added).

TABLE 3

This collection of prompts addresses the four components of fluency with basic facts. Writing about their strategies on a weekly basis engages students in self-reflection and monitoring, as well as emphasizes the importance of strategies in practicing basic facts.

Writing prompts for developing fluency with the basic facts

Appropriate strategy selection	Flexibility
• Explain how to use the "count on" strategy for 3 + 9. • What strategy did you use to solve 6 + 8? • A friend is having trouble with some of his times 6 facts. What strategy might you teach him? • Emily solved 6 + 8 by changing it in her mind to 4 + 10. What did she do? Is this a good strategy? Tell why or why not.	• How can you use 7 × 10 to find the answer to 7 × 9? • Solve 6 × 7 using one strategy. Now try solving it using a different strategy. • Emily solved 6 + 8 by changing it in her mind to 4 + 10. What did she do? Does this strategy always work?
Efficiency	**Accuracy**
• What strategy did you use to solve 9 + 3? • How can you use 7 × 7 to solve 7 × 8? • Which facts do you "just know"? For which facts do you use a strategy?	• Crystal explains that 6 + 7 is 12. Is she correct? Explain how you know. • What is the answer to 7 × 8? How do you know it is correct (how might you check it)?

Creative writing ideas that address several components

- Develop a "Face the facts" or "Ask Cougar" column (like Dear Abby) for the class. (Pick a fun name for the column that makes sense for the class, such as the school mascot.) Students send a letter about a tough fact. Rotate different students into the role of responder. The responder writes letters back, suggesting a strategy for the tough fact.
- Create a strategy rhyme (e.g., If times four is giving me trouble, I'll remember to double and double).
- Make a facts survival guide. Children prepare pages illustrating with visuals (e.g., ten frames or arrays) of how find "tough" facts.
- Write a yearbook entry to some facts (e.g., Dear 8 × 7, I …)

(See McIntosh 1997 for many more ideas).

Meaningful writing tasks can be used across grade levels and operations. **Table 3** presents a collection of writing prompts that address the four components of fluency. Having an opportunity to write about strategies on a weekly basis engages students in self-reflection and self-monitoring as well as emphasizes the importance of strategies in practicing basic facts.

4. Quizzes

You may be surprised to see this section, given the major concerns raised earlier related to timed tests, but quizzes can be used effectively to assess efficiency as well as strategy use. Ensure that students "just know" their foundational facts before moving on to derived facts. Foundational facts are so named because they can be used to generate all the other facts using a strategy. Foundational facts in addition include one- and two-more-than, combinations that make ten, and doubles. For multiplication, they include × 1, × 2, × 5, and × 10. (See Kling and Bay-Williams, in press, for a discussion of foundational facts.) From these facts, we can derive all other facts. Quiz questions (see **fig. 3a**) can be used to see if students "just know" foundational facts.

Similarly, quizzes can be used to monitor facts that come more easily to students. For example, a quiz (see **fig. 3b**) assesses if students are recognizing the commutativity of addition for one-more-than facts. Notice that these examples are shorter, not timed, and also focus on strategies. The following adaptations can enhance the effectiveness of facts quizzes:

FIGURE 3

Quizzes that focus on fluency are alternatives to timed tests.

(a) Quiz questions can be used to see if students "just know" foundational facts.

Solve these problems and tell how you solved them.

4 × 5 =	______	Check one:	______	I used this strategy: ____________________
			______	I just knew.
10 × 6 =	______	Check one:	______	I used this strategy: ____________________
			______	I just knew.
6 × 2 =	______	Check one:	______	I used this strategy: ____________________
			______	I just knew.
5 × 3 =	______	Check one:	______	I used this strategy: ____________________
			______	I just knew.
2 × 9 =	______	Check one:	______	I used this strategy: ____________________
			______	I just knew.
3 × 10 =	______	Check one:	______	I used this strategy: ____________________
			______	I just knew.
5 × 7 =	______	Check one:	______	I used this strategy: ____________________
			______	I just knew.
8 × 10 =	______	Check one:	______	I used this strategy: ____________________
			______	I just knew.

(b) A quiz assesses if students recognize the commutativity of addition for one-more-than facts. Notice that these examples are shorter, not timed, and also focus on strategies.

On completion, say to class, "Circle the row that was easier for you to solve. If they were both the same, write 'same'."

Solve these addition problems.

ROW A:	9 + 1 =	$\begin{array}{r}8\\ \underline{+\,1}\end{array}$	5 + 1 =	3 + 1 =	$\begin{array}{r}6\\ \underline{+\,1}\end{array}$
ROW B:	1 + 8 =	1 + 7 =	$\begin{array}{r}1\\ \underline{+\,4}\end{array}$	$\begin{array}{r}1\\ \underline{+\,2}\end{array}$	1 + 9 =

Adopt more flexibility and variety in how you assess students' basic fact fluency.

→ reflect and discuss

"Assessing Basic Fact *Fluency*"

Reflective teaching is a process of self-observation and self-evaluation. It means looking at your classroom practice, thinking about what you do and why you do it, and then evaluating whether it works. By collecting information about what goes on in our classrooms and then analyzing and evaluating this information, we identify and explore our own practices and underlying beliefs.

The following questions, related to "Assessing Basic Fact *Fluency*" by Gina Kling and Jennifer M. Bay-Williams, are suggested prompts to aid you in reflecting on the article and on how the authors' ideas might benefit your own classroom practice. Consider the article independently and then discuss it with your colleagues.

Think of the basic facts assessments that you are currently using with respect to the following:

- Flexibility
- Efficiency
- Appropriate strategy use
- Accuracy

1. With your current assessments, what percentage of emphasis might you assign to each of the four categories above? Is this balance what you would like it to be? If not, how might you alter your assessments to equitably address the four areas of fluency?
2. As you reflect on your students' basic facts fluency, what would you like to know more about? Which of the assessment tools from the article might help you gain this knowledge? How might you use that assessment tool?
3. Discuss your reactions to the issue of timed tests. What might you do as a teacher or leader to avoid potential negative impacts of timed tests?
4. How might we help parents better understand fluency and help their children in the areas of flexibility and selecting appropriate strategies? How might you communicate the purpose of alternative assessment tools for basic facts with your students, parents, and school leadership?

Tell us how you used Reflect and Discuss as part of your professional development. The Editorial Panel appreciates the interest and values the views of those who take the time to send us their comments. Letters may be submitted to Teaching Children Mathematics *at* ***tcm@nctm.org****. Please include Readers Exchange in the subject line. Because of space limitations, letters and rejoinders from authors beyond the 250-word limit are subject to abridgment. Letters are also edited for style and content.*

- Choose one of the problems above and write about how you solved it.
- Tell which helper fact you used the most on this quiz.
- Circle facts you "just knew." Highlight those for which you used a strategy.
- Circle facts you are sure about. Draw a square around facts that took you longer to solve.

Meaningful fact assessment for teachers and students

We recognize that using timed tests is a deeply rooted practice for measuring basic fact mastery. We hope that we have effectively made a case for *why* this practice must change and *how* to make such a change. As the NCTM Assessment Principle states, "Assessment should support the learning of important mathematics and furnish useful information to both teachers and students" (NCTM 2000, p. 11). Using the range of assessments described above accomplishes these goals, as they provide an opportunity for meaningful, targeted feedback to students that far exceeds the "right or wrong, fast or slow" feedback provided by timed testing. In fact, these assessments infuse a fifth and critical category of assessment: self-assessment. Interviews, journals, *and* quizzes on basic facts can and should encourage students to reflect on which facts and strategies they know well and which ones are tough for them. This self-assessment can be effectively followed up by having children identify and record strategies that could be used to efficiently determine the "tough" facts in the future. Over time, this self-assessment practice encourages children to instinctively apply effective strategies for challenging facts they encounter. As both teachers and students critique their growth with use of appropriate strategies, efficiency, flexibility, and accuracy, then *true* fluency with basic facts can become a reality for every child.

REFERENCES

Baroody, Arthur J. 2006. "Why Children Have Difficulties Mastering the Basic Number Combinations and How to Help Them." *Teaching Children Mathematics* 13 (August): 22–31.

Bay-Williams, Jennifer M., and Gina Kling. In press. "Enriching Addition and Subtraction

Fact Mastery through Games." *Teaching Children Mathematics*.
Boaler, Jo. 2012. "Timed Tests and the Development of Math Anxiety." *Education Week*. Online July 3, 2012.
Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf
Forbringer, Linda, and Allison J. Fahsl. 2010. "Differentiating Practice to Help Students Master Basic Facts." In *Responding to Diversity: Grades Pre-K–5,* edited by Dorothy Y. White and Julie S. Spitzer, pp. 7–22. Reston, VA: National Council of Teachers of Mathematics.
Henry, Valerie J., and Richard S. Brown. 2008. "First-Grade Basic Facts: An Investigation into Teaching and Learning of an Accelerated, High-Demand Memorization Standard." *Journal for Research in Mathematics Education* 39 (March): 153–83.
Hodges, Thomas E., Terry D. Rose, and April D. Hicks. 2012. "Interviews as RtI Tools." *Teaching Children Mathematics* 19 (August): 30–36.
Kamii, Constance, and Catherine Anderson. 2003. "Multiplication Games: How We Made and Used Them." *Teaching Children Mathematics* 10 (November): 135–41.
Kling, Gina. 2011. "Fluency with Basic Addition." *Teaching Children Mathematics* 18 (September): 80–88.
Kling, Gina, and Jennifer M. Bay-Williams. In press. "Three Steps to Mastering Multiplication Facts." *Teaching Children Mathematics*.
McIntosh, Margaret E. 1997. "500+ Writing Formats." *Mathematics Teaching in the Middle School* 2 (March/April): 354–58.
National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics.* Reston, VA: NCTM.
———. 2006. *Curriculum Focal Points for Prekindergarten through Grade 8: A Quest for Coherence.* Reston, VA: NCTM.
National Research Council (NRC). 2001. *Adding It Up: Helping Children Learn Mathematics*. Edited by Jeremy Kilpatrick, Jane Swafford, and Brad Findell. Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academies Press.
Ramirez, Gerardo, Elizabeth A. Gunderson, Susan C. Levine, and Sian L. Beilock. 2013. "Math Anxiety, Working Memory, and Math Achievement in Early Elementary School." *Journal of Cognition and Development* 14 (2): 187–202.
SBAC (Smarter Balanced Assessment Consortium). 2012. *Smarter Balanced Assessments.* http://www.smarterbalanced.org/smarter-balanced-assessments/#item
Van de Walle, John. A., Karen S. Karp, and Jennifer M. Bay-Williams. 2013. *Elementary and Middle School Mathematics: Teaching Developmentally.* Professional Development Edition. New York: Pearson.
Van de Walle, John A., Lou Ann H. Lovin, Karen S. Karp, and Jennifer M. Bay-Williams. 2014. *Teaching Student Centered Mathematics: Grades K–2.* 2nd edition. New York: Pearson.
Wiliam, Dylan. 2011. *Embedded Formative Assessment*. Bloomington, IN: Solution Tree Press.


Gina Kling, **gina.garzakling@wmich.edu**, of Western Michigan University and **Jennifer M. Bay-Williams**, **j.baywilliams@louisville.edu**, of the University of Louisville, share enthusiasm for helping children develop fluency with their basic facts. The authors thank Meaghan Gorzenski of St. Monica School and Vicky Kudwa of St. Augustine Cathedral School in Kalamazoo, Michigan, and their first-grade students for their contributions to this article.

Readers may be interested in "Research Suggests That Timed Tests Cause Math Anxiety" (p. 469), in which Jo Boaler makes a case for alternative assessment strategies.

more4U

Download a free app for your smartphone. Then scan this code to access quizzes and observation checklists appended to this article at **www.nctm.org/tcm060**.

FOCUSING IN ON

How I See Addition Facts

Jason Zimba, Student Achievement Partners

Every Saturday at 10am is "Saturday School" at our house. It's only an hour or so. Saturday School is never very long, but over time we still manage to do a range of things, including solving word problems, learning concepts, and doing exercises that build fluency and fact recall. Our materials include worksheets that I create, workbooks off the shelf, released test questions, homemade flashcards, and plenty of dice that we might use to play a math game.

Concerning flashcards in particular, a researcher in mathematics education saw this article of mine and later emailed me an important tip that I want to pass along:

> *"[Flashcards]...are a good fluency method [but] please stress that students should be spending most of their time on the cards they do not know yet or on those they know but are not fast on yet. Most drill uses many problems students know and thus is a big time waster."*

A similar message is **(emphasis added):**

> **"Organizing practice so that it focuses most heavily on understood but not yet fluent products and unknown factors can speed learning.** *To achieve this by the end of grade 3, students must begin working toward fluency for the easy numbers as early as possible."*

As Saturday School is not very long, time is always scarce. So, it is valuable to customize flashcard work to the student's current state of mastery. Some digital apps do allow you to adjust the settings so that students are focusing on the facts they need to focus on—maybe some of them even adjust themselves automatically over time. As we don't use digital apps, here's how I've tried to accomplish something similar using old-fashioned flashcards. I make no claim that my method is the best, or even better than others! But it has worked for us, and it's been fun.

Here's how it works:

- The flashcards are kept in a "piggy-bank" made from an old tissue box.
- We work at the dining table. My wife or I will draw a flashcard from the bank and show it.
- If the answer comes back "lickety-split," then the card is set aside. Otherwise, the card goes back in the bank. If there is doubt, then the card goes back in the bank.
- If the student is drawing a complete blank, then I prompt with a strategy, for example if the problem is 6 × 8 and the reaction is a blank stare, then I might prompt with "Do you know 5 × 8?" Then the student can say, "Oh right, that's 40, so 8 more is 48." Of course, the card goes back in the bank, but it has been a good just-in-time learning opportunity.

We'll do anywhere from 5 to 15 minutes of this, and at the end, the student "owns" all of the cards that have been set aside. The student highlights the known facts on a map (images below) and puts the cards into a keepsake box.

We use magnets to pin "maps" to metal shelves above the kids' desks. Each weekend, they chart new progress using a highlighter. The maps are a great way to recognize accomplishment. The kids love adding to the maps and seeing their maps fill up over time. Completing the map brings a strong sense of satisfaction and achievement. For my purposes, the map also suggests hypotheses about where prerequisite concepts might be lacking.

Since I've been doing Saturday School with my kids, I've continued to appreciate how intricate is the structure of the addition facts as they play out in the curriculum. My younger is learning some of the facts from memory and, for some others, applying the strategy of making ten.

At this stage of learning, making ten is a great strategy for a problem like 8 + 5. But the strategy doesn't help you in a problem like 12 + 3. To solve that problem, at this stage of learning, I'd say you want to:

- appreciate the place value structure as 10 + 2 + 3,
- know from memory that 2 + 3 = 5, and
- understand the meaning of teen numbers so that 10 + 5 = 15.

In general, each region of the $a + b$ map has its own story. This graphic shows the map with different regions color-coded. The map extends beyond the basic addition facts to include all whole-number sums with results less than or equal to 20.

0+0	1+0	2+0	3+0	4+0	5+0	6+0	7+0	8+0	9+0	10+0	11+0	12+0	13+0	14+0	15+0	16+0	17+0	18+0	19+0	20+0
0+1	1+1	2+1	3+1	4+1	5+1	6+1	7+1	8+1	9+1	10+1	11+1	12+1	13+1	14+1	15+1	16+1	17+1	18+1	19+1	
0+2	1+2	2+2	3+2	4+2	5+2	6+2	7+2	8+2	9+2	10+2	11+2	12+2	13+2	14+2	15+2	16+2	17+2	18+2		
0+3	1+3	2+3	3+3	4+3	5+3	6+3	7+3	8+3	9+3	10+3	11+3	12+3	13+3	14+3	15+3	16+3	17+3			
0+4	1+4	2+4	3+4	4+4	5+4	6+4	7+4	8+4	9+4	10+4	11+4	12+4	13+4	14+4	15+4	16+4				
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0+6	1+6	2+6	3+6	4+6	5+6	6+6	7+6	8+6	9+6	10+6	11+6	12+6	13+6	14+6						
0+7	1+7	2+7	3+7	4+7	5+7	6+7	7+7	8+7	9+7	10+7	11+7	12+7	13+7							
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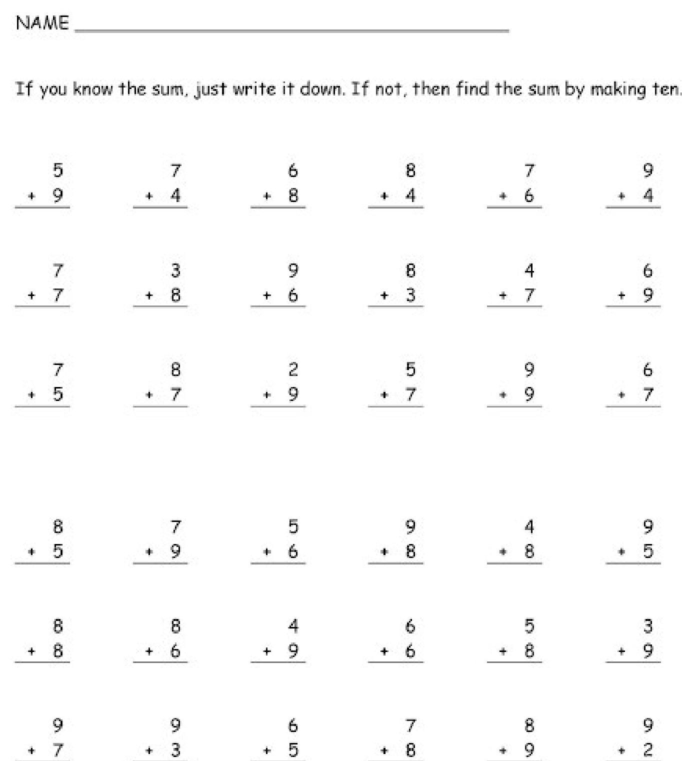
0+18	1+18	2+18																		
0+19	1+19																			
0+20																				

Here is a key to the colors:

6+1	Counting on
6+0	Property of 0
2+3	Addition within 10
4+6	Partners of 10
8+5	Making 10
10+7	Meaning of teen numbers
13+4	Place value computation
10+10	Link between count sequence and place value

I use this map to orient myself toward my kids' learning as they progress toward fluency with addition facts and knowing them from memory. For example, the image below shows the worksheet I made for a recent Saturday School. (Downloadable PDF here.) You'll see that the worksheet has 36 sums for practice—precisely the 36 sums that are coded yellow above. So this worksheet is for an intense practice day on making ten. Over the previous few Saturdays, we had established that the partners of 10 (magenta) were down cold, that there was fluency within 10 (red), and that the structure of the teen numbers was well understood (green). These are the key prerequisites for making ten. (Another is the ability to use properties of addition where helpful.)

The goal for single-digit sums is to know them from memory (2.OA.1). Until the goal is reached, students approach a problem like 7 + 4 as just that: a problem.


NAME ______________________

If you know the sum, just write it down. If not, then find the sum by making ten.

5 + 9	7 + 4	6 + 8	8 + 4	7 + 6	9 + 4
7 + 7	3 + 8	9 + 6	8 + 3	4 + 7	6 + 9
7 + 5	8 + 7	2 + 9	5 + 7	9 + 9	6 + 7
8 + 5	7 + 9	5 + 6	9 + 8	4 + 8	9 + 5
8 + 8	8 + 6	4 + 9	6 + 6	5 + 8	3 + 9
9 + 7	9 + 3	6 + 5	7 + 8	8 + 9	9 + 2

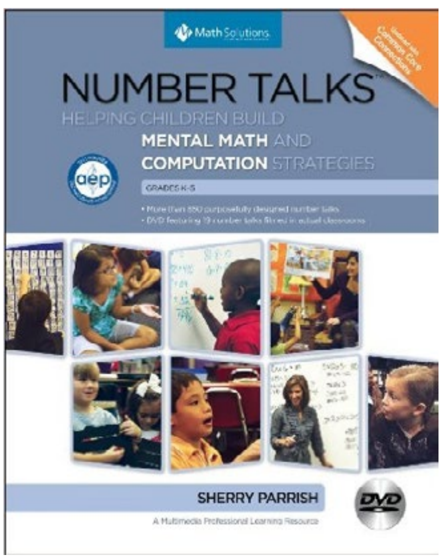
I hope the map is interesting to study and productive to think about. I know that it helps me to lay a strong foundation for future learning by focusing me and my kids on the best strategy or way of thinking about each type of addition problem.

FOCUSING IN ON

Number Talks

Tessa Ziser, CMT Editorial Panelist

I LOVE TO READ AND LOVE to learn, which is probably why teaching is my passion and career. Over the summer, I often pick up books about pedagogy, classroom management, assessments, or any other reading material that helps me grow as an educator. I also read books over the school year, but, as all teachers know, that time is always limited. Last fall, a colleague recommended a book to me while I was attending the CCTM conference in September. Because of the time of year and the cost of the book, it took me a couple months to finally purchase and read it. I am so glad I did, but wish it had been part of my library years ago. *Number Talks: Helping Children Build Mental Math and Computation Strategies* by Sherry Parrish is now an essential part of my professional library as an elementary mathematics teacher.



At the time of reading *Number Talks*, I had been frustrated with the traditional method of practicing and testing number fluency: timed tests. These tests made many of my students anxious, were taken out of context, and often did not actually tell me how fluent my students were. Some students excelled at these tests and some students were very good at memorizing the facts before they moved on to the next test. However, fluency is different from memorization. Parrish explains that our students need to be able to compute with accuracy, efficiency, and flexibility. She goes on to define these terms: "*Accuracy* denotes the ability to produce an accurate answer; *efficiency* refers to the ability to choose an appropriate, expedient strategy for a specific computation problem; and *flexibility* means the ability to use number relationships with ease in computation" (2010, p. 5). *Number Talks* helped me move beyond simply asking my students to memorize their math facts, and guide them into a stronger conceptual understanding of numbers and how they relate to each other.

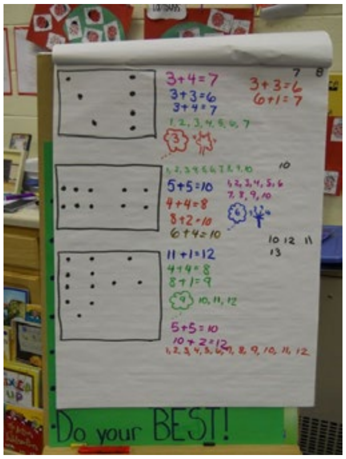
As the title suggests, Parrish introduces "number talks" as a way for students to practice computational fluency and explain their thinking. These talks support teachers in both of the Mathematics Teaching Practices from *Principles to Actions* that are highlighted in this issue of *Colorado Mathematics Teacher*: *build procedural fluency from conceptual understanding, and facilitate meaningful math discourse.*

So what exactly are number talks? Parrish explains:

> The introduction of number talks is a pivotal vehicle for developing efficient, flexible, and accurate computation strategies that build upon the key foundational ideas of mathematics such as composition and decomposition of numbers, our system of tens, and the application of properties. Classroom conversations and discussions around purposefully crafted computation problems are at the very core of number talks. These are opportunities for the class to come together to share their mathematical thinking. The problems in a number talk are designed to elicit specific strategies that focus on number relationships and number theory. Students are presented with problems in either a whole- or small-group setting and are expected to learn to mentally solve them accurately, efficiently, and flexibly. By sharing and defending their solutions and strategies, students are provided with opportunities to collectively reason about numbers while building connections to key conceptual ideas in mathematics. A typical classroom number talk can be conducted in five to fifteen minutes (2010, p. 5).

In other words, number talks are simply a structure to use in your classroom that help you facilitate conversations around number computations.

After explaining the what and why of number talks, Parrish goes on to explain, in detail, how to prepare for them. The most beneficial portion of this chapter for me was the instruction on how to record student thinking. It helped me be more comfortable and confident when first implementing this structure in my own classroom. The rest of Parrish's book contains more specific information about number talks in the K–2 and 3–5 classrooms. These chapters include many examples of different sets of equations that can be used to support good discussion. This is an especially valuable resource because it organizes the different sets based on what strategy you would like the students to be practicing and discussing.



As I was implementing number talks last year, this book was always on hand so that I could quickly and easily find equations to introduce during our daily talks.

As the new school year begins, I look forward to starting number talks in my classroom on the first day of school. Convinced more than ever that timed math fact tests are not beneficial for my students (for more information and research on this topic, see Jo Boaler's article "Fluency Without Fear"), number talks will play a prominent role in helping students practice and me, as the teacher, assess fact fluency. I am excited to make them a daily part of our routine and to support students in a conceptual understanding of computations and in discourse that helps create mathematical thinkers. Sherry Parrish's *Number Talks* will be a well-used book in my 3rd grade classroom.

IN THE CLASSROOM

ARCs: Activities with Rigor and Coherence

Janet Oien, NCTM Representative

Last summer I had the opportunity to work with other teachers from across the country as part of the NCTM Classroom Resource Development Team (CRDT). CRDTs support the work of the newly formed NCTM Classroom Resources Committee (CRC). The CRC's vision is to create an online professional learning community that is the go-to site for mathematics teachers.

We first met in July at the NCTM headquarters in Reston, VA. It was exciting arriving at the office, then being escorted to a large boardroom to begin our week of work. We were warmly welcomed with personalized name tags and folders of information to guide our work. The initial group of 20 teachers were divided into grade level bands (elementary, middle and high school). I was part of the high school team. Our time was divided between whole group and small group discussions. During whole group discussions, topics included our goals, updates and feedback. Small group sessions were for our writing teams. We had full access to all NCTM resources like articles and journals, and if we needed something, they got it for us. It was pretty cool!

Our first major task was to create ARCs. What are ARCs you ask? ARC stands for Activities with Rigor and Coherence; this acronym captures our goals as we were writing them. Each ARC is a series of lessons that addresses a mathematical topic and demonstrates the vision of *Principles to Actions: Ensuring Mathematical Success for All.* They scaffold effective teaching and support enactment of the eight Mathematics Teaching Practices articulated in *Principles to Actions*, as well as the instructional guidance set forth in *5 Practices for Orchestrating Productive Mathematics Discussions*. ARCs integrate a wide array of NCTM resources to optimize opportunities for learning, including Illuminations and Student Explorations in Mathematics. ARCs also include community features that offer opportunities for social interaction, engaging in online discussions with other math educators, posting a comment, and giving feedback with ratings and reviews.

We started with *Illuminations* lesson plans that needed revamping,then generated new topics based on needs. The high school team completed two ARCs during the week—The Law of Sines and Cosines, and Coding and Decoding.

> *On a side note - If you don't know what Illuminations lessons are, you should check them out! Illuminations, a free resource on the NCTM website, has lessons and interactives. Teachers can search by grade level or topic. One of my go-to interactives is the dynamic paper teachers can use to make many different types of graphs, grids, polar graphs, net figures, etc. There is also an enjoyable KenKen interactive on the site, along with several great elementary interactives. More about Illuminations can be found at* http://illuminations.nctm.org/

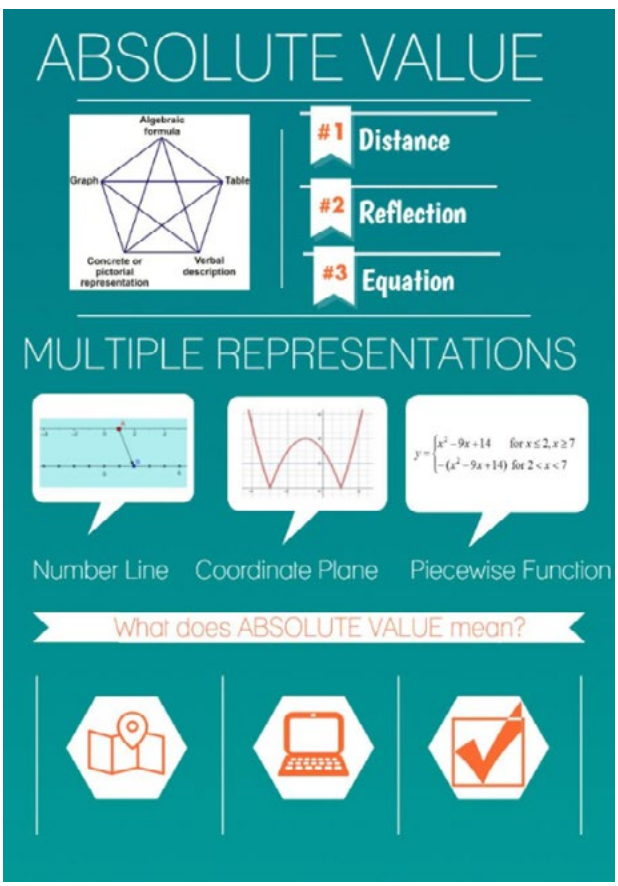
Back to ARCs - In February the CRDTs met once again to continue our work. There were a few new faces this time and not everyone from the summer work session was able to make it. So we began by briefly reviewing the goals of the CRDT, then jumped right into brainstorming topics for the ARCs we would write that weekend. We searched for topics that teachers have a difficult time teaching conceptually and/or don't have many good resources. The high school group ended up choosing absolute value, triangle congruence via transformations, regressions, and graphing trigonometric functions.

As we worked together in teams creating ARCs, we realized how much we were learning from each other as well as the writing process. This got us thinking about how to get more people involved in reviewing and creating Activities with Rigor and Coherence so that more NCTM members could benefit from the process. We felt like part of the value in these activities was not just in the products produced and used in the classroom, but in also in the process of creating, providing feedback about, and sharing ideas around each topic.

I was on the team working with the **Absolute Value** ARC. Of the three lessons we have been working on, two are ready and made available on the website. At April's NCTM national convention in San Francisco, we offered a soft release of the ARCs website. While some of our work is still a work in process, selected activities at each grade level are now available for teachers to search for and use, complete with handouts and interactives. A road map is provided for accessing lessons.

Below is a visual of the Absolute Value Landing Page for **High School, Algebra**. Explore the meaning of absolute value as a distance from zero using a variety of visual representations.

Accessing the page, you will see a description and overview of the ARC. Some ARCs include a storyboard poster that captures the ideas of the lessons. On our storyboard, we tried to capture the idea of multiple representations of absolute value, so it includes a double number line, coordinate plane, and piecewise function representation.



Activities with Rigor and Coherence: Multiple Representations of Absolute Value

Grade Level: High School Algebra

Description: Explore the meaning of absolute value as a distance from zero using a variety of visual representations.

Storyboard: This ARC aims to create contexts for students to experience a variety of visual representations that reinforce the definition of absolute value. Explorations connecting back to elementary concepts and progressing in complexity provide intentional transitions for building deeper meaning. Representations considered in subsequent activities include the following topics:

- Number line - as the (positive) distance from zero.
- Double number lines - as functions in one-dimension.
- Coordinate Plane - as a graph of a piecewise function.

Some of the lessons, such as ours, contain audio clips of CRDT members discussing topics, strategies, or ideas for a lesson.

What makes the ARCs so fantastic is the ability for our members to now engage with one another. Check out the comments, ratings and reviews. One of the CRC members gave a great analogy related to online recipes that I would like to share: "There are thousands of sites to get recipes, but I like allrecipes.com because of how many comments there are. No site will give a comprehensive list for perfect recipes. What keeps you coming back to it is how the community *adds value* to it. You can read about people's modifications of the recipe on how to make it better, see pictures, the most popular recipes, etc."

Similarly, NCTM envisions this form of collaboration happening with its resources. They need you to get involved! NCTM would love for you to try the activities out in your classroom and provide comments: modifications, general thoughts, opportunities for differentiation, and reflections after implementation.

The following ARCs are currently on the NCTM website:

Counting Strategies Landing Page

Kindergarten, Counting & Cardinality. Students focus on understanding the relationship between numbers and quantities by counting, producing, and constructing the numbers 0-10 as well as developing 5 as a benchmark.

Growing Patterns Landing Page

Grade 5, Operations & Algebraic Thinking. Students build and analyze geometric growing patterns, determine rules, and write expressions to describe the growth models.

Discovering Area Relationships Landing Page

Grade 6, Geometry. Students discover the area formulas for triangles, parallelograms, and trapezoids, and then apply them to find areas of irregular figures.

Absolute Value Landing Page

High School, Algebra. Explore the meaning of absolute value as a distance from zero using a variety of visual representations.

Barbie Bungee Landing Page

High School, Statistics. Students collect and analyze data to help them predict the longest bungee cord that Barbie can use safely.

Triangle Congruence

Middle School, Geometry. Students discover the area formulas for triangles, parallelograms, and trapezoids. They apply formulas to find areas of irregular figures.

Go to http://www.nctm.org/ARCs/ for activities that continue to be posted and check back often for comments and feedback from other teachers.

IN THE CLASSROOM

Mathematical Modeling: Part 1 of Series

Catherine Martin, CCTM Past President and NCTM Board Member

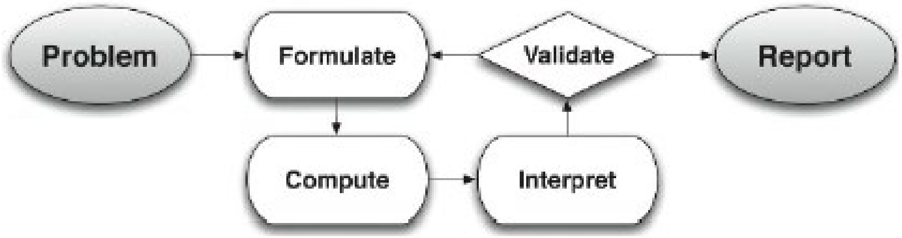
Modeling, *model*, and *models* are words that appear multiple times in the Colorado Academic Standards (Common Core State Standards). They can be found across all grade levels and strands in phrases such as visual fraction model, concrete models, mathematical models, modeling situations, area models, probability models, physical models, model a linear relationship, descriptive modeling, modeling context, model periodic phenomena. These examples, however, fail to capture the essence of mathematical modeling as called for in Math Practice 4: Model with Mathematics. Furthermore, these examples do not capture the description of modeling in the conceptual category of modeling in the high school standards. More recently, the Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME), released in April 2016, provides specific guidance and support to teachers as they incorporate the practice of mathematical modeling into classroom instruction.

Modeling Defined

Beginning with Math Practice 4 (excerpted below), key student actions for engaging in this Practice are:

> Mathematically proficient students can **apply the mathematics they know to solve problems arising in everyday life, society, and the workplace**. Mathematically proficient students who can apply what they know are **comfortable making assumptions and approximations** to simplify a complicated situation, realizing that these may need revision later. They are able to **identify important quantities in a practical situation and map their relationships using such tools** as diagrams, two-way tables, graphs, flowcharts and formulas. They can **analyze those relationships mathematically to draw conclusions.** They routinely **interpret their mathematical results in the context of the situation** and **reflect on whether the results make sense, possibly improving the model if it has not served its purpose.** (CCSSI, 2010, p. 5)

Further, the modeling cycle (CCSSO, p. 72) in the high school standards provides a visual representation of the modeling cycle and actions captured in Math Practice 4.



In addition, the GAIMME report provides a succinct definition of mathematical modeling that captures the essence of Math Practice 4 and the modeling cycle, "Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena" (GAIMME, 2016, p. 8). This definition clearly communicates that an essential element is a real-world context or situation. In addition, mathematics is used in the process to explore and investigate the real-world context or situation. Math Practice 4 describes specific student actions to be used as they explore and investigate the context or situation. Students make assumptions or approximations, identify important quantities and relationships, analyze the relationships and make conclusions and interpret these in terms of the context. If the conclusions do not make sense in terms of the context, students begin the exploration and investigation again and refine their model in doing so.

An important caveat to note is that a word problem is not necessarily a modeling problem. A modeling problem, as described in the GAIMME report, has the following characteristics:

- relevant and important context for students,
- open in the beginning: multiple entry points and question definition,
- open in the middle: multiple mathematical approaches,
- open in the end: multiple solutions,
- questions are inspired and assumptions are required, and
- useful solutions.

Examples of Modeling Problems YOU Can Use and Help CCTM Colleagues Delve Deeper

So what are examples of modeling problems? Let's look at a task from grades K–5, grades 6–8, and high school. In each example, consider how the task is open in the beginning, in the middle, and at the end. What information would students need? What assumptions would they need to make?

- Grades K–5 Task: Class Party

Our class is planning a party. Several students have suggested we buy pretzels for the party. How many bags of pretzels should we buy? (adapted from GAIMME)

- Grades 6–8 Task: Stray Dogs

Your class has decided to recommend to your city council that stray dogs be fed. How much dog food would the city council need to buy to feed all the stray dogs in your city for one year? (adapted from GAIMME)

- High School Task: Gas Station Problem

Is it worth it to drive a little farther to a gas station that has cheaper gas? What would make it worth it? (Pahler, 2016)

Teachers are encouraged to use one of these problems with students and send student work or reflections on using these problems to Cathy Martin (cathy_martin@dpsk12.org). The next article in this series will dive deeper into GAIMME and recommendations for classroom implementation of mathematical modeling.

References

National Governors Association (NGA) & Council of Chief State School Officers (CCSSO). (2010). Common Core State Standards Initiative (CCSSI).

Garfunkel, S. & Montgomery, M. (EDs). (2016). GAIMME: Guidelines for Assessment & Instruction in Mathematical Modeling Education. Consortium for Mathematics and Its Applications (COMAP) & Society for Industrial and Applied Mathematics (SIAM).

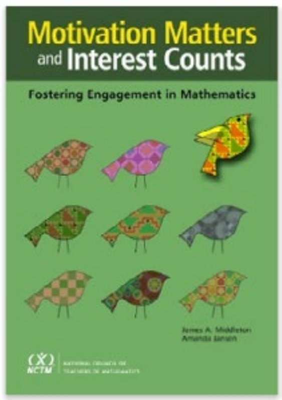
Pahler, L. (April 2016). Guidelines for Assessment & Instruction in Mathematical Modeling Education. Presentation at NCTM.

BOOK REVIEW

Motivation Matters and Interest Counts

Reviewed by Cassie Gannett, CCTM Treasurer

Motivation Matters and Interest Counts by James A. Middleton and Amanda Jansen (2011) is a must have for any educator looking for ways to engage students in learning mathematics. The compilation of research-based strategies acknowledges all types of learners and how they may see themselves as mathematicians. Middleton and Jansen begin with the presupposition that we are all innately inquisitive and motivated to learn: beginning with the child who is exploring the world for the first time and continuing through the adolescent discovering what type of adult he/she will become.



This book is practical and useful for educators because it offers case studies of real students who are either motivated or uninterested and offers everyday solutions to capture the student's attention. Every chapter begins with a story of a student or students as they interact with mathematics education, either with them reflecting on their own experiences and feelings or with a narrator in a classroom situation. This situation is then referred to throughout the chapter as the author describes various strategies that teachers can use to support student motivation and interest in mathematics.

One such chapter is "Motivation is Social." This chapter is prefaced by a story of a 7th grade student who begins to experience success in the math class and tells a friend, "Math's my thing." The vignette continues to outline the way that a teacher can introduce a task about raising funds for a charity. The students begin to discuss the charity of their choice. Although the discussion was not grounded in mathematics, the author illustrates that by allowing the students to consider the task in terms of a context that is of interest to them, they are more motivated to complete the task. The students then work in groups to complete the task, collaborating and supporting one another in their learning. Throughout the chapter, the author discusses ways that students interact with mathematics socially in terms of how they see themselves and how others might see them, with considerations for the positive implications of this relatedness. While offering several strategies to promote the positive effects of students learning socially, the author concludes the chapter with the statement, "Rather than assuming that socializing should be kept to a minimum in the mathematics classroom, we can instead capitalize on students' interests in socializing to promote their learning" (p. 83).

This book is a valuable resource for middle and high school teachers who often encounter students who have already developed views about themselves as mathematicians and the expectations for learning. In order to motivate students to continue persevering and working through challenges, the strategies in this book are practical and useful. They help us work towards our goal of having every student enter our classroom excited for the learning of the day.

CDE CORNER

Raymond Johnson, CDE Mathematics Content Specialist

All Students, All Standards Institute

Last June in Keystone, almost 200 educators across all subject areas described by the Colorado Academic Standards came together for three days to focus on the kinds of instructional shifts needed to support high-quality standards and curriculum implementation. For mathematics, I was privileged to work with more than 20 great teachers representing 13 different districts, and together we worked on developing, delivering, and reflecting on lessons that targeted particular teaching practices.

Participants at the All Students, All Standards summer institute held in Keystone (L to R):Jeff Spencer (Lake County), Karl Remsen (Lake County), Raymond Johnson (CDE), Janelle Stanton (Boulder Valley), Cari Pettyjohn (Greeley), Roslyn Wikoff (Greeley), Brad Summerville (Metro State), Debbie Moore (Archuleta County), Scott Fokken (Briggsdale), Jaime Akers (Archuleta County), Pam Brenner (Boulder Valley), Ginger Walker (Springfield), Douglas Freese (Englewood), Samantha Aragon (Metro State), Tammy Walter (Colorado Springs 11), Candace Grant (Douglas County), Caitlin Price (Archuleta County), Shannon Collings-Helfer (Colorado Springs 11), Jim Freund (Boulder Valley) (Not pictured: Phillip Hutcherson, Colorado Springs 11, and Dan Van der Vieren, Weldon Valley)

If you followed math educators on blogs and Twitter in June, you might have seen some great discussions about the value of shared lesson plans. Dan Meyer might have summed it up best with a quote from Dwight Eisenhower: "In preparing for battle I have always found that plans are useless, but planning is indispensable." With this in mind, our collective goal was not to produce sets of instructions for what content to teach and how, but rather to understand and communicate **how** the lesson was planned, **why** we made the pedagogical choices we made, and through **reflection** to judge if those choices led to intended outcomes. Teachers spent an afternoon in grade band groups planning a lesson then delivered that lesson the next day to teachers (from other subject areas)—who took on the role of students!

To add some structure— but not too much—to the lesson planning, I gave participants a "menu" of tools to work with. Some of these tools are well known, which helped teachers build off some of what they already knew and were using. The menu was organized into four parts:

1. Planning Guidance: Participants were asked to choose one of the following resources to help guide their planning:

- 5 Practices for Orchestrating Productive Mathematics Discussions by Smith & Stein,
- The "Thinking Through a Lesson Protocol" from the article, "Thinking Through a Lesson: Successfully Implementing High-Level Tasks" by Smith, Bill, & Hughes,
- Lesson Planning Tool from Student Achievement Partners, or
- CDE lesson plan guide focused on teacher process and metacognition.

2. Instructional Model: To structure the delivery of the lesson, participants chose one of the following instructional models:

- Launch, Explore, Summarize model from the Connected Mathematics Project,
- 3-Act Task model from Dan Meyer and others, or
- 5E Instructional Model from BSCS.

Lecture and "I Do, We Do, You Do" are also instructional models, and each has its place and purpose in the classroom. For this conference, however, we focused on the kinds of student-centered instructional models that students should experience the majority of the time.

Jeff Spencer of Leadville helps another participant during the high school math practice teaching lesson.

3. Teaching Practice: Each group was asked to focus on improving aspects of their instructional practice, which we took from the eight Mathematics Teaching Practices described by NCTM's Principles to Actions:

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

Although teachers engage in multiple practices on a continual basis, math teachers at the conference focused on improving and getting feedback about just one or two. Participants could also consider high-leverage practices from the more comprehensive list developed by the TeachingWorks project.

4. Reflection: Teachers were asked to use a common reflection tool at the end of the lesson. Some of the participants were instructional coaches, so their reflection tools doubled as observation tools. With each tool, teachers were encouraged to include some aspect of student work, such as student group work or an exit ticket. Some of the tools used were:

- SERP 5 x 8 Card (a simple observation tool designed for principals without math expertise),
- "Practical Measures" as described by the Carnegie Foundation for the Advancement of Teaching,
- Instructional Practice Guide CoachIng Tool from Student Achievement Partners, and
- Bank of reflection questions for teachers and students developed by CDE.

(L to R) Tammy Walter (Colorado Springs 11), and Caitlin Price and Debbie Moore (Archuleta County 50) guide participants through a practice elementary math lesson.

This fall, participating teachers will work to develop and deliver lessons in their own classrooms, and then share their processes with us all. The format of these lessons is a work in progress, but they will stay true to the goals of sharing the planning and the reflection, and not just a lesson plan. If you would like to know more, let me know at Johnson_R@cde.state.co.us.

PAEMST Finalists Selected

The Presidential Awards for Excellence in Mathematics and Science Teaching (PAEMST) are the nation's highest honors for K–12 teachers of mathematics and science (including computer science). Awardees serve as models for their colleagues, inspiration to their communities, and leaders in the improvement of mathematics and science education.

Although we are still awaiting the announcement by the White House of the 2014 and 2015 Presidential Awardees, I am proud to announce Colorado's two PAEMST award finalists for the year 2016:

- Lindsey Gunderson-Hahn is a 1st grade teacher at Field Elementary in Littleton Public Schools. She received her Bachelor of Arts from the University of Northern Colorado and has a Masters of Education from Regis University. She has eight years of teaching experience and has previously taught grades 4 and 5. Lindsey is dedicated to bringing high-quality mathematics learning to a high-poverty school, and through coaching, collaboration, and professional learning she has developed exemplary skill in facilitating classroom discourse that engages all of her students.
- Sarah Ogier teaches 4th grade and coaches other teachers at Field Elementary in Littleton Public Schools. She received her Bachelor of Arts from Westmont College in California, has a Master of Arts in Curriculum and Instruction from the University of Colorado Denver, and also a Graduate Certificate in Mathematics Teacher Leadership from the University of Northern Colorado. Sarah has 10 years of teaching experience and has been coaching other teachers since 2013. Because of her high quality teaching, her classroom is continually visited by interested teachers and leaders.

I hope you can join me this fall at the Annual CCTM Awards Reception on September 22 to congratulate these teachers and to wish them well in the prestigious (and apparently slow) awardee selection process.

NCTM BOARD REPORT

Strategic Priorities

Catherine Martin, CCTM Past President and NCTM Board Member

THE NCTM STRATEGIC PRIORITIES for our work as an organization are: **access and equity; advocacy; curriculum, instruction and assessment; professional development; research; and technology.** Recent or future activities underway to support these strategic priorities are described below.

- To better understand the unique needs of our Canadian members, the NCTM Board met in Toronto, Canada in February. Canadian mathematics colleagues joined the Board to engage in conversation on what is happening in mathematics education in Canada—from both a teacher and policy perspective—and to discuss how NCTM can better support Canadian members.

- NCTM hosted an equity meeting on March 24, 2016 with NCTM affiliates—AMTE (Association of Mathematics Teacher Educators), ASSM (Association of State Supervisors of Mathematics), BBA (Benjamin Banneker Association), NASGEm (North American Study Group on Ethnomathematics), NCSM, TODOS: Mathematics for ALL, WME (Women and Mathematics Education)—to discuss collaborative actions to promote access, equity and excellence for marginalized students. Priority actions were identified, including some actions that are already underway.

- The 2017 NCTM Annual Meeting will be in San Antonio, April 5–8, 2017.

- NCTM is working on a new design of "regional conferences" to include the traditional regional conference and an Innov8 conference design. In November 2016 (St. Louis), NCTM will host the first Innov8 Conference: "Engaging the Struggling Learner" (with three themes: developing instruction and assessments for multi-tiered systems of support, supporting productive struggle, and motivating the learner who struggles). For more information on 2016 Innov8, go to: https://www.nctm.org/innov-8about/. In addition, two regional conferences will be held, one in Phoenix and one in Philadelphia. The 2017 Innov8 Conference (Las Vegas) will focus on access, equity and empowerment (agency, identity, and social justice). With NCTM conferences around the county, consider attending one that meets your goals.

- In response to concerns around computer science credit and teacher evaluation, NCTM has produced two new position statements that are now available on the NCTM website: "Computer Science and Mathematics Education" and "Evaluation of Teachers of Mathematics."

- NCTM hosted a State Advocacy Conference for Mathematics Education with state department and NCTM state affiliate leaders from 12 states. The focus was to increase knowledge about effective advocacy regarding high-quality mathematics teaching and learning, to develop relationships within and across states to support effective advocacy, and to commit to take action. Look for the NCTM-Hunt Institute Video Series on the NCTM website.

- Classroom Resources Committee (CRC) is creating ARCs (Activities with Rigor and Coherence). These resources are aligned with the effective teaching practices in *Principles to Actions*, focus on a specific mathematics topic, support the Standards for Mathematical Practice, and demonstrate the five practices for orchestrating productive mathematics discussions. The first ARCs are posted on NCTM website under classroom resources. Illuminations are also posted under classroom resources.

CONFERENCES AND PROFESSIONAL DEVELOPMENT

2016 CCTM Conference: Building a Community of Math Excellence

Leigh Ann Kudloff, Conference Chair

The Colorado Council of Teachers of Mathematics' upcoming annual conference, "Building a Community of Math Excellence," will be held on September 22–23, 2015 at the Denver Mart, and will offer learning opportunities for everyone in our mathematics community, including elementary, secondary, and pre-service teachers, coaches, professors, and math leaders. Conference sessions will align to learning strands for those interested in STEM, lessons and activities, coaching, special populations, growth mindset, effective teaching practices, assessment, and the Standards for Mathematical Practice. Through collaboration with TODOS, the 2016 CCTM Conference will also explore diversity issues and will provide a strand focusing on mathematics for ALL learners.

Thursday

The Conference kicks off on Thursday, September 22nd, with an administrator and district leader workshop from 12:00–3:30 p.m. A box lunch is included for those that pre-register, and participants will have an opportunity to network during lunch. This session will be facilitated by Joanie Funderburk and Cathy Martin, President and Past President of CCTM, and will address specific needs of administrators, district leaders, math coaches/specialists, and others who support classroom teachers. The session, "Curricular Materials Review: A Professional Learning Opportunity," will allow leaders to explore rubrics for evaluating mathematics curricular materials, glean ideas for incorporating free online resources with existing curriculum, and gain an understanding of how curricular review processes provide a professional learning structure for teachers.

Following the leader pre-session on Thursday, Nora Ramirez and José Franco will lead the teacher pre-session (4:00–6:30 p.m.), exploring ways to make mathematics accessible and equitable for all students. Be sure to pre-register for this engaging session for teachers of mathematics. The annual Colorado Mathematics Teaching Awards ceremony and reception follows at 6:30 p.m. We hope you will join us in honoring our award winners and nominees while celebrating excellence in Colorado classrooms.

Friday

On-site registration and check-in for Friday's conference will begin at 7:00 a.m. Dr. Rochelle Gutiérrez, professor of Mathematics Education in Curriculum and Instruction at the University of Illinois at Urbana-Champaign, will be our featured keynote speaker, and will share her research on equity in mathematics education. Over 120 breakout sessions will be offered, with multiple sessions appropriate for teachers of all grade bands. Vendors will be located in the exhibit hall throughout the conference, and a session of inspiring IGNITE! talks and door prize giveaways will end your conference experience. Your registration fee includes a lunch voucher that can be spent among several onsite catering options, allowing you to make the most of your conference time.

Hotel Information and Conference Registration

For those requiring an overnight stay close to the conference, a block of rooms has been reserved at the DoubleTree Westminster (located on the Boulder Turnpike at Sheridan) for a conference rate of $115/night, including a breakfast buffet and a free shuttle to and from the conference venue. Make your reservation today. When the block of rooms is sold out, the conference rate may not be available.

Conference registration has begun! We look forward to seeing you soon!

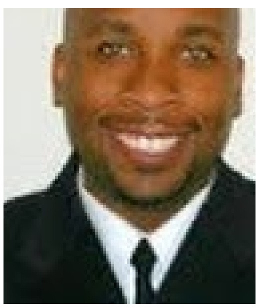
MEET THE BOARD

Welcome New Members

Sandie Gilliam

In previous CMT issues, you have been introduced to some of your math colleagues who volunteer to serve by being on the board of Colorado Council of Teachers of Mathematics. Three of our new board members are Kevin Duren, Jill Buecking, and Erica Marlys Hastert.

Kevin Duren (Vice President), graduated from Colorado State University in 1992 with a bachelor's degree in mathematics and landed his first teaching position at Adams 50 in Denver. During this time, he was awarded a National Science Foundation scholarship to continue his education and went back to school at the University of Northern Colorado to complete his master's degree in mathematics. Kevin taught math and computer programming for from 1993–2002. Formerly both an assistant principal and a high school principal, Kevin is currently an executive director of secondary schools and director of math achievement in the Widefield School District (Colorado Springs).



Duren has three children who have all graduated from college. His sons are working as engineers in the Denver metro area, and his daughter is currently finishing her nursing degree at the University of Northern Colorado. Along with his love of teaching and learning, he is a private pilot, musician, and amateur computer programmer. His love of mathematics only has been accentuated by his use and application of math throughout his career.

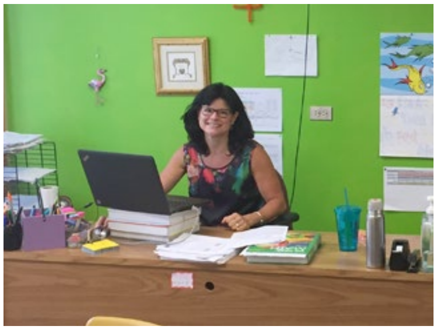
Jill Buecking (Region 6 Representative) grew up in New Jersey and lived in Jackson Hole, New York City, and Denver before moving to Eagle County. She earned a Bachelor of Arts in Philosophy and Religion from Colgate University, a Master of Arts in Teaching from Colorado College, and a Doctor of Education focusing on Leadership for Educational Equity from University of Colorado, Denver.

Jill taught kindergarten through 4th grade in Cherry Creek Schools before becoming a learning and instruction specialist in Eagle County. She loves the work she is doing supporting PK–12 math learning. The best part is she learns every day from the teachers in Eagle County Schools!

The last five years of Jill's career have brought amazing opportunities to work with Math Recovery and to learn about how students make sense of numbers. These experiences have contributed to Jill's beliefs about equity, access, and the importance of mindset in mathematics.

"I believe that CCTM's role is to help build a community of confident and competent mathematics educators in Colorado. Students in Colorado deserve teachers who are passionate about mathematics and have the belief that all students are capable of learning. I am committed to ensuring teachers are provided with quality professional learning opportunities and collaboration, whether it be face to face or virtually. I believe that CCTM provides educators in Colorado with these things and I am excited to represent Region 6 on the board."

Erica Marlys Hastert (MAA Representative) has been a Colorado math student and teacher almost her entire life. Erica graduated from Aurora Central High School in 1984, and studied math (probability and statistics emphasis) at CU-Denver, earning her BA; math and computer science at the University of Colorado, Denver to obtain her MS degree; and Quantitative Research Methods within the Educational Psychology program at the University of Denver, resulting in her Ph.D. in 1998.



After short stints in both an actuarial career (Security Life of Denver and William M. Mercer, Inc.) and statistical research with ACT, Inc. (Iowa City, IA), Erica returned to her true love of teaching math in 1993 at Arapahoe Community College, where she served in many leadership roles, including department chair, faculty senate president, and Phi Theta Kappa advisor. Erica was also an online instructor and associate dean for math and science for Colorado Community Colleges Online.

With her heart remaining in the classroom, she is now studying for her secondary math certification while teaching math at the Early College of Arvada, a college preparatory middle and high school that partners with CU-Denver to provide up to 59 credit hours of college credit to high school juniors and seniors.

Erica has served as president of Colorado Mathematical Association of Two Year Colleges, vice-chair of the Rocky Mountain section of MAA, and is currently serving as the Local Events Coordinator for the national American Mathematical Association of Two Year Colleges (AMATYC) conference in Denver this coming November.

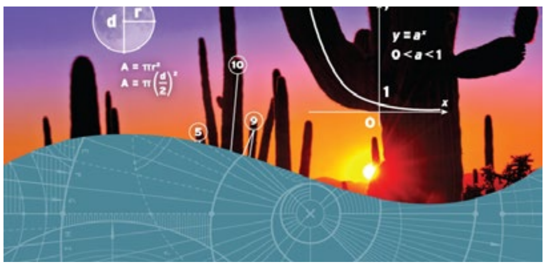
"I'm new to NCTM and CCTM, but this organization has come highly recommended to me for many years by my colleagues. I have always been active in MAA and AMATYC, and with my new high school teaching career, joining CCTM as the MAA liaison seemed the perfect opportunity. I am very excited to explore, share and contribute to the many resources that CCTM has to offer, and to connect what Colorado math teachers are doing with the goals of the MAA. I look forward to meeting and working with wonderful new colleagues, and embarking on this next chapter of my career!"

Fast Connections

UPCOMING NCTM CONFERENCES *CLOSE* TO COLORADO

Can't make the CCTM conference in September but want to deepen your professional knowledge? Consider attending:

October 26–28, 2016. NCTM Regional Conference, *Great Math at Your Doorstep*, in **Phoenix, AZ**



Strands include:

- Creating Mathematical Minds: Effective Teaching & Learning for the 21st Century (PTA Strand)
- Mathematics Beyond the Classroom
- Linking Assessment and Instruction
- Mathematics Across the Curriculum
- Building Mathematics Culture and Capacity
- Equity: Developing Mathematical Understanding for All

April 5–8, 2017, NCTM Annual Conference, *Creating Communities and Cultivating Change*, in **San Antonio, TX**

Strands include:

- Access and Equity: Teaching Mathematics with an Equity Stance
- Assessment: A Tool for Purposeful Planning and Instruction
- Building Conceptual and Procedural Understanding
- Professionalism: Learning Together as Teachers
- Teaching, Learning, & Curriculum: Best Practices for Engaging Students in Productive Struggle
- The "M" in STEM/STEAM
- Tools and Technology: Using Technology to Effectively Teach and Learn Mathematics

2016 COLORADO PROFESSIONAL OF THE YEAR AWARD

Congratulations go out to Mary Pittman, former CCTM CDE Representative upon her receiving the Colorado Council for Learning Disabilities (CCLD) 2016 Colorado Professional of the Year Award and the prestigious Floyd G. Hudson Outstanding Service Award. Go Mary!